Subject CS2

2025 Study Guide

Contents

1 Introduction

This Study Guide has been created to help you navigate your way through Subject CS2. It contains useful information you will need before starting to study Subject CS2 for the 2025 exams. You may also find it useful to refer to throughout your studies.

Further information on study skills can be found on our website at **ActEd.co.uk**.

Please read this Study Guide carefully before reading the Course Notes, even if you have studied for some actuarial exams before.

Before you start

When studying for the Institute and Faculty of Actuaries' exams, you may need:

- a copy of the **Formulae and Tables for Examinations of the Institute and Faculty of Actuaries, 2nd Edition (2002)** – these are referred to simply as the *Tables*
- a **scientific calculator** and/or **software package** to help with calculations.

The *Tables* are available from the Institute and Faculty of Actuaries' eShop. Please visit **actuaries.org.uk**.

2 Subject sequencing and contents

2.1 Links to other subjects

Associateship Qualification

This subject assumes that the student is competent with the material covered in Subject CS1 – Actuarial Statistics – and the required knowledge for that subject.

Subjects CM1 and CM2 – Actuarial Mathematics for Modelling and Economic Modelling – apply the material in this subject to actuarial and financial modelling.

Fellowship Qualification

Topics in this subject are further built upon in Subject SP1 – Health and Care Principles, Subject SP7 – General Insurance Reserving and Capital Modelling Principles, Subject SP8 – General Insurance Pricing Principles and Subject SP9 – Enterprise Risk Management Principles.

2.2 Subject contents

There are five parts to the Subject CS2 course. The parts cover related topics and are broken down into chapters. At the end of each part there are assignments testing the material from that part.

The following table shows how the parts and chapters relate to each other. The final column shows how the chapters relate to the days of the regular tutorials. This table should help you plan your progress across the study session.

Part	Chapter	Title	No of pages	X Asst	Y Asst	Tutorial – 5 days
1	1	Stochastic processes	38	X1	Y1	1
	2	Markov chains	78			
	3	The two-state Markov model and the Poisson model	41			
	4	Time-homogeneous Markov jump processes	76			
2	5	Time-inhomogeneous Markov jump processes	60	X2		2
	6	Survival models	41			
	7	Estimating the lifetime distribution	61			
3	8	Proportional hazards models	46	X3		3
	9	Exposed to risk	36			
	10	Graduation and statistical tests	66			
	11	Methods of graduation	31			
	12	Mortality projection	63			
4	13	Time Series 1	75	X4	Y2	4
	14	Time Series 2	75			
	15	Loss distributions	57			
	16	Extreme value theory	56			
5	17	Copulas	57	X5		5
	18	Reinsurance	48			
	19	Risk models 1	39			
	20	Risk models 2	43			
	21	Machine learning	83			

3 Syllabus

The Syllabus for each subject is produced by the Institute and Faculty of Actuaries. It includes information to support the study of this subject. The Syllabus will guide you through what you need to learn, the application of learning, as well as the skills that you need to develop.

Students can use the Syllabus as a guide for learning and development. We recommend that you use the Syllabus as an important part of your study.

3.1 Aim

The aim of Subject CS2 is to provide a grounding in mathematical and statistical modelling techniques that are of relevance to actuarial work, including stochastic processes and survival models and their application.

3.2 Topics and topic weightings

This subject covers the following topics:

1. Random variables and distributions for risk modelling (20%)
2. Time series (20%)
3. Stochastic processes (25%)
4. Survival models (25%)
5. Machine learning (10%)

The topic weighting percentage noted alongside the topics is indicative of the volume of content of a topic within the subject and therefore broadly aligned to the volume of marks allocated to this topic in the examination. For example, if a topic is 20% of the subject then you can expect that approximately 20% of the total marks available in the examination paper will be available on that topic.

Students should ensure that they are well prepared across the entire syllabus and have an understanding of the principal terms used within the course.

3.3 Objectives

The detailed syllabus objectives for Subject CS2 are given below. To the right of each objective are the chapter numbers in which the objective is covered in the ActEd course. The relevant individual syllabus objectives are also included at the start of each course chapter.

1. **Random variables and distributions for risk modelling (20%)**

Statistical distributions suitable for modelling the variables and risks that arise within insurance contracts:

1.1 Loss distributions, with and without risk sharing (Chapters 15 and 18)

1.1.1 Properties of the statistical distributions that are suitable for modelling individual and aggregate losses.

1.1.2 Concepts of excesses (deductibles) and retention limits.

1.1.3 Operation of simple forms of proportional and excess of loss reinsurance.

1.1.4 Calculate the distribution and corresponding moments of the claim amounts paid by the insurer and the reinsurer in the presence of excesses (deductibles) and reinsurance.

1.1.5 Estimate the parameters of a failure time or loss distribution when the data is complete, or when it is incomplete, using maximum likelihood and the method of moments.

1.1.6 Fit a statistical distribution to a data set and calculate appropriate goodness-of-fit measures.

1.2 Compound distributions and their applications in risk modelling (Chapters 19 and 20)

1.2.1 Construct models appropriate for short-term insurance contracts in terms of the numbers of claims and the amounts of individual claims.

1.2.2 Major simplifying assumptions underlying the models in 1.2.1.

1.2.3 Compound Poisson distribution and apply the result that the sum of independent random variables, each having a compound Poisson distribution, also has a compound Poisson distribution.

1.2.4 Mean, variance and coefficient of skewness for compound binomial, compound Poisson and compound negative binomial random variables.

1.2.5 Loss distributions for both the insurer and the reinsurer after the operation of simple forms of proportional and excess of loss reinsurance where underlying losses take the forms given in 1.2.4.

1.3 Introduction to copulas (Chapter 17)

1.3.1 Characterise a copula as a multivariate distribution function that is a function of the marginal distribution functions of its variates and explain how this allows the marginal distributions to be investigated separately from the dependency between them.

1.3.2 Meaning of the terms 'dependence or concordance', 'upper and lower tail dependence', and state in general terms how tail dependence can be used to help select a copula suitable for modelling particular types of risk.

1.3.3 Know the form and characteristics of the Gaussian copula and the Archimedean family of copulas.

1.4 Introduction to extreme value theory (Chapter 16)

1.4.1 Recognise extreme value distributions, suitable for modelling the distribution of severity of loss and their relationships.

1.4.2 Calculate various measures of tail weight and interpret the results to compare the tail weights.

2. **Time series** **(20%)**

Statistical concepts for modelling, fitting and forecasting data that is indexed by time.

2.1 Understand the core concepts underlying time series models (Chapters 13 and 14)

2.1.1 General properties of stationary, $I(0)$, and integrated, $I(1)$, univariate time series.

2.1.2 Stationary random series.

2.1.3 Stationary random series with a filter applied.

2.1.4 Know the notation for backwards shift operator, backwards difference operator, and the concept of roots of the characteristic equation of time series.

2.1.5 Concepts and basic properties of autoregressive (AR), moving average (MA), autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) time series.

2.1.6 Concepts and properties of discrete random walks and random walks with normally distributed increments, both with and without drift.

2.1.7 Basic concept of a multivariate autoregressive model.

2.1.8 Cointegrated time series.

2.1.9 Univariate time series models with the Markov property and rearranging a univariate time series model as a multivariate Markov model.

2.2 Applications of time series models (Chapters 13 and 14)

2.2.1 The processes of identification, estimation and diagnosis of a time series, the criteria for choosing between models, and the diagnostic tests that may be applied to the residuals of a time series after estimation.

2.2.2 Other non-stationary, non-linear time series models.

2.2.3 Simple applications of a time series model, including random walk, autoregressive and cointegrated models as applied to security prices and other economic variables.

2.2.4 Develop deterministic forecasts from time series data, using simple extrapolation and moving average models, applying smoothing techniques and seasonal adjustment when appropriate.

3. **Stochastic processes** **(25%)**

Application of Markov models to model time-indexed risk and claim data arising primarily in insurance and other appropriate business-related scenarios:

3.1 Stochastic processes. (Chapter 1)

3.1.1 Stochastic processes, in particular a counting process.

3.1.2 Understand whether a stochastic process:

- operates in continuous or discrete time
- has a continuous or a discrete state space
- is a mixed type

3.1.3 Applications of mixed processes.

3.1.4 The Markov property in the context of a stochastic process and in terms of filtrations.

3.2 Understand and apply a Markov chain. (Chapter 2)

3.2.1 Essential features of a Markov chain model.

3.2.2 State the Chapman-Kolmogorov equations that represent a Markov chain.

3.2.3 Calculate the stationary distribution for a Markov chain in simple cases.

3.2.4 Understand and apply systems of frequency-based experience rating in terms of a Markov chain.

3.2.5 Time-inhomogeneous Markov chain models and simple applications.

3.2.6 Markov chains as a tool for modelling and how they can be simulated.

3.3 Define and apply a Markov process. (Chapters 4 and 5)

3.3.1 Essential features of a Markov process model.

3.3.2 Poisson process, derive the distribution of the number of events in a given time interval, derive the distribution of inter-event times, and apply these results.

3.3.3 Kolmogorov equations for a Markov process with time-independent and time/age-dependent transition intensities.

3.3.4 Kolmogorov equations in simple cases.

3.3.5 Simple survival models, sickness models and marriage models in terms of Markov processes and describe other simple applications.

3.3.6 Kolmogorov equations for a model where the transition intensities depend not only on age/time, but also on the duration of stay in one or more states.

3.3.7 Sickness and marriage models in terms of duration-dependent Markov processes and describe other simple applications.

3.3.8 Markov jump processes as a tool for modelling and how they can be simulated.

4. Survival models (25%)

Description, estimation and use of statistical models for the time until an event occurs:

4.1 Concepts of survival models.

4.1.1 Model of lifetime or failure time from age x as a random variable. (Chapter 6)

4.1.2 Consistency condition between the random variable representing lifetimes from different ages. (Chapter 6)

4.1.3 Distribution and density functions of the random future lifetime, the survival function, the force of mortality or hazard rate, and derive relationships between them. (Chapter 6)

4.1.4 Understand the actuarial symbols ${}_tp_x$ and ${}_tq_x$ and derive integral formulae for them. (Chapter 6)

4.1.5 Gompertz and Makeham laws of mortality. (Chapter 6)

4.1.6 Curtate future lifetime from age *x* and its probability function. (Chapter 6)

4.1.7 Understand the symbols e_x and $\mathring{e}_x$ and derive an approximate relation between them. Define the expected value and variance of the complete and curtate future lifetimes and derive expressions for them. (Chapter 6)

4.1.8 Two-state model of a single decrement and comparing its assumptions with those of the random lifetime model. (Chapter 3)

4.2 Understand the estimation procedures for lifetime distributions.

4.2.1 The various ways in which lifetime data may be censored. (Chapter 7)

4.2.2 Estimation of the empirical survival function in the absence of censoring, and what problems are introduced by censoring. (Chapter 7)

4.2.3 The Kaplan-Meier (or product-limit) estimator of the survival function in the presence of censoring, computing it from typical data and estimating its variance. (Chapter 7)

4.2.4 The Nelson-Aalen estimator of the cumulative hazard rate in the presence of censoring, computing it from typical data and estimating its variance. (Chapter 7)

4.2.5 Models for proportional hazards and how these models can be used to estimate the impact of covariates on the hazard. (Chapter 8)

4.2.6 The Cox model for proportional hazards, deriving the partial likelihood estimate in the absence of ties, and the asymptotic distribution of the partial likelihood estimator. (Chapter 8)

4.3 Derive maximum likelihood estimators for transition intensities. (Chapters 3 and 4)

4.3.1 Identify an observational plan in respect of a finite number of individuals observed during a finite period of time, and define the resulting statistics, including the waiting times.

4.3.2 Understand the likelihood function for constant transition intensities in a Markov model of transfers between states given the statistics in 4.3.1.

4.3.3 Identify maximum likelihood estimators for the transition intensities in 4.3.2 and state their asymptotic joint distribution.

4.3.4 State the Poisson approximation to the estimator in 4.3.3 in the case of a single decrement.

4.4 Transition intensities dependent on age (exact or census). (Chapter 9)

4.4.1 Dividing the data into homogeneous classes, including subdivision by age and sex.

4.4.2 The principle of correspondence and its fundamental importance in the estimation procedure.

4.4.3 Specify the data needed for the exact calculation of a central exposed to risk (waiting time) depending on age and sex.

4.4.4 Calculate a central exposed to risk given the data in 4.4.3.

4.4.5 Understand how to obtain estimates of transition probabilities.

4.4.6 Identify the assumptions underlying the census approximation of waiting times.

4.4.7 The concept of the rate interval.

4.5 Graduation and graduation tests (Chapters 10 and 11)

4.5.1 Statistical tests of the comparison of crude estimates with a standard mortality table testing for:

- the overall fit
- the presence of consistent bias
- the presence of individual ages where the fit is poor
- the consistency of the 'shape' of the crude estimates and the standard table.

For each test describe:

- the formulation of the hypothesis
- the test statistic
- the distribution of the test statistic using approximations where appropriate
- the application of the test statistic.

4.5.2 Reasons for graduating crude estimates of transition intensities or probabilities, and the desirable properties of a set of graduated estimates.

4.5.3 How to test for smoothness of a set of graduated estimates.

4.5.4 The process of graduation by the following methods, and the advantages and disadvantages of each:

- parametric formula
- standard table
- spline functions

(The candidate will not be required to carry out a graduation.)

4.5.5 How the tests in 4.5.1 should be amended to compare crude and graduated sets of estimates.

4.5.6 How the tests in 4.5.1 should be amended to allow for the presence of duplicate policies.

4.5.7 Carry out a comparison of a set of crude estimates and a standard table, or of a set of crude estimates and a set of graduated estimates.

4.6 Mortality projection (Chapter 12)

4.6.1 Approaches to the forecasting of future mortality rates based on extrapolation, explanation and expectation, as well as their advantages and disadvantages.

4.6.2 Lee-Carter, age-period-cohort, and *p*-spline regression models for forecasting mortality.

4.6.3 Use an appropriate computer package to apply the models in 4.6.2 to a suitable mortality data set.

4.6.4 Identify the main sources of error in mortality forecasts.

5. **Machine learning** **(10%)**

5.1 Elementary principles of machine learning. (Chapter 21)

5.1.1 The bias/variance trade-off and its relationship with model complexity.

5.1.2 Cross-validation to evaluate models on unseen data, and to estimate hyperparameters.

5.1.3 Understand how regularisation can be used to reduce overfitting in highly parameterised models.

5.1.4 The use of software to apply supervised learning techniques to solve regression and classification problems.

5.1.5 The use metrics such as precision, recall, F_1 score and diagnostics such as the ROC curve and confusion matrix to evaluate the performance of a binary classifier.

5.1.6 Unsupervised learning techniques (principal component analysis and K -means clustering) to reduce data dimensionality, identify latent substructure and detect anomalies.

4 Core Reading

This section explains the role of the Core Reading and how it links to the Syllabus, supplementary ActEd text and the examination.

4.1 Core Reading

The Core Reading has been produced by the Institute and Faculty of Actuaries. It supports students in their learning and development of this subject by providing information and explanation of the topics and objectives in the Syllabus.

The Core Reading is updated annually to reflect any changes to the Syllabus and current practice, as well as for continuous improvement.

The current version of the Core Reading is up-to-date as of 31 May 2024. It references the version of any legislation, standards, professional guidance, *etc* as of this date. Any known upcoming changes to the references are noted where relevant in the Core Reading.

Accreditation

The Institute and Faculty of Actuaries would like to thank the numerous people who have helped in the development of the material contained in the Core Reading.

Further reading

A list of additional resources to support candidate learning and development for this subject can be found on the Module pages on the Institute and Faculty of Actuaries' website:

actuaries.org.uk/curriculum/

4.2 Links to the Syllabus

Each part of the Core Reading relates directly to the Syllabus.

The relevant syllabus objectives are included at the start of each chapter for reference.

The Core Reading supports coverage of the Syllabus in helping to ensure that both depth and breadth are re-enforced.

4.3 Links to the examination

Examiners can set questions based on any area of the Syllabus within any examination sitting and will consider and draw from the Core Reading when setting examinations questions.

Students will be expected to apply the Core Reading to scenarios and questions proposed by the examiners.

The exams in April and September 2025 will be based on the Syllabus and Core Reading as at 31 May 2024. We recommend that you always use the up-to-date Core Reading to prepare for the exams.

Past papers indicate to students how the examiners apply the Core Reading. The Examiners' Reports provide further insight as to how students answered the questions and how marks were awarded.

4.4 ActEd text

The Core Reading deals with each syllabus objective and covers what is needed to pass the exam, and the Subject CS2 Course Notes include the Core Reading in full, integrated throughout the course.

However, the tuition material that has been written by ActEd enhances it by giving examples and further explanation of key points. Here is an excerpt from some ActEd Course Notes to show you how to identify Core Reading and the ActEd material. **Core Reading is shown in this bold font.**

In the example given above, the index *will* fall if the actual share price goes below the theoretical ex-rights share price. Again, this is consistent with what would happen to an underlying portfolio.

After allowing for chain-linking, the formula for the investment index then becomes:

$$I(t) = \frac{\sum_i N_{i,t} P_{i,t}}{B(t)}$$

This is ActEd text

This is Core Reading

where $N_{i,t}$ is the number of shares issued for the *i*th constituent at time *t*;

$B(t)$ is the base value, or divisor, at time *t*.

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5 Skills and assessment

5.1 Technical skills

Subjects CS1 and CS2 are very mathematical and have relatively few questions requiring wordy answers.

5.2 Exam skills

In each examination, students will be expected to demonstrate, through their answers, that they have knowledge of, can apply and use higher order skills in this subject:

- Knowledge will be demonstrated through answering questions that assess understanding of that knowledge as well as through questions that ask for the application of relevant knowledge to scenarios.
- Application will be demonstrated through answering questions that assess the ability to identify and apply relevant concepts and skills to solve problems (both numerical and non-numerical).
- Higher order skills will be demonstrated through questions that will assess the ability to use relevant knowledge, concepts and skills to solve problems, draw appropriate conclusions, and make meaningful and appropriate comments on those conclusions.

In the CS subjects, the approximate split of assessment across the three skill types is:

- Knowledge – 20%
- Application – 65%
- Higher Order skills – 15%.

The Institute and Faculty of Actuaries use command verbs (such as 'Define', 'Discuss' and 'Explain') to help students to identify what the question requires. The examination can be composed of questions drawing from any part of the syllabus and using any command verb.

The Institute and Faculty of Actuaries has produced guidance on 'Command verbs used in the Associate and Fellowship examinations', to help students to understand what each command verb is asking them to do.

You can find the relevant document on the Institute and Faculty of Actuaries' website at:

actuaries.org.uk/qualify/prepare-for-your-exams

5.3 Assessment

Assessment is in the form of two timed, online examinations:

- Paper A is 3 hours and 20 minutes and consists of a number of questions of varying marks, for which the answers must be constructed and typed in Microsoft Word
- Paper B is 1 hour and 50 minutes and consists of a number of questions of varying marks, for which the answers must typically be constructed using R and typed using Microsoft Word.

This includes reading time, as well as the time taken for students to download and/or print the question paper.

In order to pass this subject, both Paper A and Paper B must be sat within the same sitting, and a combined mark of a pass achieved.

5.4 Further information

The Institute and Faculty of Actuaries has produced a number of documents, which it advises students to read and understand. In particular, the:

- Qualification Handbook, which contains information on studying and preparing for exams, as well as available support and resources
- Examinations Handbook, which contains practical assistance on how to sit an Institute and Faculty of Actuaries' examination
- Assessment Regulations document, which includes rules on eligibility, entry and conduct during an online assessment.

The Qualification Handbook can be found at:

actuaries.org.uk/qualify/student-and-associate-exam-news/qualification-handbook

The Examinations Handbook and Assessment Regulations document can be found at:

actuaries.org.uk/qualify/my-exams/ifoa-exams

IMPORTANT NOTE: These documents may be updated and re-published in the weeks leading up to each exam session. It is important that you keep up-to-date with any changes and developments.

6 ActEd study support

An overview of ActEd's products and services, and guidance on how to choose the best ones for you, can be found on our website at **ActEd.co.uk/productguide**.

6.1 Summary of ActEd products and services

Detailed descriptions of all ActEd's products and services can be found on our website at **ActEd.co.uk**. However, the specific products and services available for Subject CS2 include:

- Course Notes
- Paper B Online Resources (PBOR), including the Y Assignments
- X Assignments – five assignments:
  - X1, X2, X3: 80-mark tests (you are allowed 2¾ hours to complete these)
  - X3, X4: 100-mark tests (you are allowed 3¼ hours to complete these)
- Y Assignments – two assignments:
  - Y1, Y2: 100-mark tests (you are allowed 1¾ hours to complete these)
- Series X Marking
- Series Y Marking
- Online Classroom – over 150 tutorial units
- Flashcards
- Sound Revision
- Revision Notes – twelve A5 booklets
- ASET (2020-23 papers) – four years of exam papers, covering the period April 2020 to September 2023
- Mini-ASET – covering the April 2024 exam paper
- Mock Exam – one 100-mark test for the Paper A examination and a separate 100-mark test for the practical Paper B exam
- Additional Mock Pack (AMP) – two additional 100-mark Paper A tests and two additional 100-mark Paper B tests
- Mock Exam Marking
- Marking Vouchers.

Products are generally available in both paper and eBook format. Visit **ActEd.co.uk** for full details about available eBooks, software requirements and restrictions.

6.2 Tuition

The following tutorials are typically available for Subject CS2:

- Regular Tutorials (five full days / ten half days)
- Block Tutorials (five days)
- a Preparation Day for the practical exam.

Tutorials are typically available both face-to-face and live online.

Full details are set out in our *Tuition Bulletin*, which is available on our website at **ActEd.co.uk**.

6.3 Questions and queries

From time to time you may come across something in the study material that is unclear to you.

Our online discussion forum at **ActEd.co.uk/forums** (or use the link from our home page at **ActEd.co.uk**) is dedicated to actuarial students so that you can get help from fellow students on any aspect of your studies from technical issues to study advice. ActEd tutors visit the site regularly to ensure that you are not being led astray and we also post other frequently asked questions from students on the forum as they arise.

If you are still stuck, then you can send queries by email to the Subject CS2 email address **CS2@bpp.com**, but we recommend that you try the forum first. We will endeavour to contact you as soon as possible after receiving your query but you should be aware that it may take some time to reply to queries, particularly when tutors are running tutorials. At the busiest teaching times of year, it may take us more than a week to get back to you.

If you have many queries on the course material, you should raise them at a tutorial or book a personal tuition session with an ActEd tutor. Please email **ActEd@bpp.com** for more details.

6.4 Feedback

If you find an error in the course, please check the corrections page of our website (**ActEd.co.uk/paper_corrections.html**) to see if the correction has already been dealt with. Otherwise, please send the details via email to the Subject CS2 email address **CS2@bpp.com**. Our tutors work hard to ensure that the courses are as clear as possible and free from errors.

ActEd also works with the Institute and Faculty of Actuaries to suggest developments and improvements to the Syllabus and Core Reading. If you have any comments or concerns about the Syllabus or Core Reading, these can be passed on via ActEd. Alternatively, you can send them directly to the Institute and Faculty of Actuaries' Examination Team by email to **memberservices@actuaries.org.uk**.

7 General information and support

7.1 Safeguarding

We want you to feel comfortable within our learning environment and safe in the knowledge that if you ever needed support, you know where to go.

If you need support, please contact BPP's Safeguarding team at **safeguarding@bpp.com** or for urgent concerns call 07464 542 636.

Additional information can be found at **ActEd.co.uk/learningsupport**.

7.2 BPP learning support

BPP's Learning Support team offers a wide range of support for all students who disclose a learning difficulty or disability. This support is accessible to all ActEd students free of charge.

Please contact BPP's Learning Support team at **LearningSupport@bpp.com** for more information.

Additional information can be found at **ActEd.co.uk/learningsupport**.

7.3 The Prevent Duty

The Prevent Duty is to protect people from radicalisation and being drawn into extremist views and terrorism. As a Government-regulated training provider, ActEd has a duty to ensure that our learners are well informed and stay safe, and to empower our students to know what to look for and when to report concerns.

Please report any concerns to a tutor or email **safeguarding@bpp.com** or for urgent concerns call 07464 542 636.

More information is available at:

- **ActEd.co.uk/learningsupport**
- **officeforstudents.org.uk/advice-and-guidance/student-wellbeing-and-protection/counter-terrorism-the-prevent-duty/**